SOLUTIONS TO THE "'PAUSE AND THINK"' QUESTIONS

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Abstract. This document presents solutions to the "'Pause and Think"' questions in the course reading materials about discrete mathematics.

1. Questions about sets

Page 27, first question: Solution: $S \in P(S)$.
Page 27, second question: Solution:

$$P(S) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

Page 48, first question: Solution: $A \cap B$.
Page 48, second question: Solution: $A \cup B$.
Page 52, first question: Solution: No, as the inclusion is obvious from the definitions.
Page 52, second question: Solution: No, as the inclusion is obvious from the definitions.

2. Questions about functions

Page 60: Solution: Yes.
Page 64: Solution: It makes sense only when f is a function.
Page 71, first question: Solution: The number of elements in $f(X)$ could be any integer between 1 and n.
Page 71, second question: Solution: The number of elements in $f^{-1}(Y)$ cannot be determined.
Page 77: Solution: The domain and codomain are R, the range is $[-1, +1]$.
Page 83: Solution: This question refers to the set expressions on Page 79.

In this case, $f(f^{-1}(Y)) = Y$ and $X = f^{-1}(f(X))$.

In this case, $f(X \cup X') = X \cup X'$ and $f(X \cap X') = f(X) \cap f(X')$.

In this case, the equality in the remaining two expressions of course remains.

Page 89: Solution: The answer is Yes if set A is the domain of both f and g.
Page 94, first question: Solution: a straightline parallel to the X-axis.
Page 94, second question: Solution: a straightline going through the original and has the same angel with the x-axis and y-axis.

Page 100: Solution: The set representation of the function for the unary complement operation is:

$$\begin{array}{ll} f: & P(U) \to P(U) \\ f: & A \mapsto U - A \end{array}$$

The set representation of the function for the binary intersection operation is:

$$\begin{array}{ll} f: & P(U) \times P(U) \to P(U) \\ f: & (A, B) \mapsto A \cap B. \end{array}$$

Page 106, first question: Solution: No. This is because $\cos(x)$ could take the value 0, while $\log(0)$ is not defined.

Page 106, second question: Solution: Yes, beacuse the range of the function log is a subset of the domain of the function exp.

Page 110: Solution: hgf is a function from R to R, and

$$\begin{aligned} (hgf)(x) &= (hg)(f(x)) \\ &= (hg)(x+1) \\ &= h(g(x+1)) \\ &= h((x+1)^2) \\ &= \frac{1}{1+(x+1)^4}. \end{aligned}$$

Page 116: Solution: 3!=6 one-to-one functions.

Page 122: Solution: No. For example, let $A = \{0, 1, 2, 3\}$, and $B = C = \{0, 1\}$. Define a function from A to B by

$$f(x) = x \bmod 2$$

and let $g(x) = x$ be the function from B to C. Clearly, gf is not 1-1, but g is 1-1.

Page 129: Solution: No. Note that $1 \in Z_+$, but it does not have a preimage in Z.

Page 135: Solution: No. For example, let $A = B = \{0, 1, 2, 3\}$, and $C = \{0, 1, 2, 3, 4\}$. Define a function from A to B by

$$f(x) = x$$

and let $g(x) = x$ be the function from B to C. Clearly, gf is not onto, but f is onto.

Page 141: Solution: $3! = 6$ 1-1 correspondences.

Page 148: Solution: No. For example, let $A = B = \{0, 1, 2, 3\}$, and $C = \{0, 1, 2, 3, 4\}$. Define a function from A to B by

$$f(x) = x$$

and let $g(x) = x$ be the function from B to C. Clearly, gf is not a bijection, but f is a bijection.

Page 155, first question: Solution: Yes.

Page 155, second question: Solution: No, because both $(a, 1) \in f^{-1}$ and $(a, 3) \in f^{-1}$.

Page 162, first question: Solution: Yes.
Page 162, second question: Solution: Yes.
Page 166: Solution: No, as the preimage of y may not be unique.
Page 173: Solution:

$$I_A f^{-1} = f^{-1}, \quad f^{-1} I_B = f^{-1}.$$