COMP 2211 Midterm Exam - Spring 2022 - HKUST

Date: April 2, 2022 (Saturday)
Time Allowed: 2 hours, 2:00–4:00 pm
Instructions:

1. This is a closed-book, closed-notes examination.
2. There are **7** questions on **25** pages.
3. Write your answers in the space provided.
4. All programming codes in your answers must be written in the Python version as taught in the class.
5. For programming questions, unless otherwise stated, you are **NOT** allowed to define additional classes, helper functions and use global variables, nor any library functions not mentioned in the questions.

Student Name	SOLUTIONS AND MARKING SCHEME
Student ID	
Email Address	

For T.A. Use Only

Problem	Topic	Score
1	True/False Questions	/ 15
2	Python Fundamentals	/ 16
3	Conditional Probability and Bayes Classifier	/ 11
4	K-Nearest Neighbors	/ 14
5	K-Means Clustering	/ 12
6	Perceptron	/ 20
7	Perceptron and Multilayer Perceptron	/ 12
	Total	/ 100

Problem 1 [15 points] True/False Questions

Indicate whether the following statements are true or false by putting **T** or **F** in the given table. You get 1.5 points for each correct answer.

(a) Machine learning is a sub-field of artificial intelligence.

(b) Tensorflow is easy to use and less flexible than Keras.

(c) Suppose we wish to calculate $P(B|e_1, e_2)$ and we have no conditional independence information. Having $P(e_1, e_2), P(B), P(e_1, e_2|B)$ are sufficient for the calculation.

(d) Training a K-nearest neighbors classifier takes more computational time than applying it.

(e) K-nearest neighbors cannot be used for regression.

(f) Consider D-fold cross-validation. A higher number of folds (i.e. larger value of D), the estimated error will be lower on average.

(g) K-means clustering algorithm is guaranteed to converge.

(h) Consider a two-class classification problem. Suppose we have trained a perceptron model on a linearly separable training set, and now we get a new labeled data point which is correctly classified by the model, and far away from the decision boundary. If we add this new point to our earlier training set and re-train with the same set of initial weights and bias, the learnt decision boundary will be changed for sure.

(i) Gradient descent is usually **NOT** guaranteed to converge at global minimum.

(j) If the learning rate is too small, gradient descent may take a very long time to converge and computationally expensive.

Question	Answer (T/F)
(a)	T
(b)	F
(c)	T
(d)	F
(e)	F
(f)	T
(g)	T
(h)	F
(i)	T
(j)	T

Marking scheme:

- 1.5 points for each correct answer.

Problem 2 [16 points] Python Fundamentals

For each of the Python expressions below, write the output when the expression is evaluated. If the output is an empty array, write "Empty Array". If an error occurs, write "Error".

(a) [5 points] Consider the following NumPy arrays:

```
import numpy as np
A = np.array([10, 20, 30, 40, 50, 60, 70, 80, 90])
B = np.array([ [0, 1, 2, 3],
               [4, 5, 6, 7],
               [8, 9, 10, 11],
               [12, 13, 14, 15] ])
```

(i) `print(A[2:6:3])`

Answer: `[30 60]` *# 1 point*

(ii) `print(B[0:2, 1:3])`

Answer:

```
[[1 2]
 [5 6]]
```
 # 1 point

(iii) `print(A[-1:-3])`

Answer:

```
Empty Array
```
 # 1 point

(iv) `print(B[::-1])`

Answer:

```
[[12 13 14 15]
 [ 8  9 10 11]
 [ 4  5  6  7]
 [ 0  1  2  3]]
```
 # 1 point

(v) `print(B[:3,2:])`

Answer:

```
[[ 2  3]
 [ 6  7]
 [10 11]]
```
 # 1 point

Remark:

- 0.5 point will be deducted if the answers are in raw number format without [](i.e. 1 2 5 6). Because in this case, the answer is no longer in array type, and we cannot distinguish whether the answer is a 2d array or 1d array. This deduction only performs once for a(i) and a(ii).

(b) [2 points] What is the output of the following code?

```
import numpy as np
X = np.array([2,2,0,2,2,0,1,1,0,1,1,0])
Y = X[X != 0]
print(Y[::2])
```

Answer:

```
[2 2 1 1]            # 2 points
```

Remark:

- 0.5 point will be deducted if the answers are in raw number format without [](i.e. 1 2 5 6). Because in this case, the answer is no longer in array type, and we cannot distinguish whether the answer is a 2d array or 1d array. This deduction only performs once for for a(i) and a(ii).

(c) [9 points] Given the following code which computes the distance between each training point in X_train and each test point in X_test using a nested loop over both the training data and test data.

```
import numpy as np

def compute_distances_nested_loops(X_train, X_test):
    num_test = X_test.shape[0]
    num_train = X_train.shape[0]
    distances = np.zeros((num_test, num_train))
    # --- BLOCK TO REWRITE ---
    for i in range(num_test):
      for j in range(num_train):
        distances[i,j] = np.sqrt(np.sum(np.square(X_test[i,:]-X_train[j,:])))
    # --- BLOCK TO REWRITE ---
    return distances

Train = np.array([[0,1], [1,2], [2,3], [3,4]])
Test = np.array([[5,6], [7,8]])
print(compute_distances_nested_loops(Train, Test))
```

```
# Output:
# [[7.07106781 5.65685425 4.24264069 2.82842712]
#  [9.89949494 8.48528137 7.07106781 5.65685425]]
```

Rewrite the block of code between the comment lines `# --- BLOCK TO REWRITE ---` using no explicit loops in the space provided.

Hints:

- To compute the distance between training data point $(0, 1)$ and test data point $(5, 6)$, we do

$$dist = (0-5)^2 + (1-6)^2$$

- Expand it

$$\begin{aligned} dist &= 0^2 - 2(0)(5) + 5^2 + 1^2 - 2(1)(6) + 6^2 \\ &= 0^2 + 1^2 + 5^2 + 6^2 - 2(0)(5) - 2(1)(6) \\ &= 0^2 + 1^2 + 5^2 + 6^2 - 2((0)(5) + (1)(6)) \end{aligned}$$

You may find the following functions useful for this question.

- Dot product of two arrays:
  `numpy.dot(a, b)`
  - It returns the product of matrix multiplication.
- Return the element-wise square of the input
  `numpy.square(x)`
  - x is the input data
- Return the sum of array elements over a given axis.
  `numpy.sum(a, axis=None)`
  - a is an array with elements to sum.
  - axis: None or int or tuple of ints. axis = 0 means along the column and axis = 1 means working along ther row.
- Return the non-negative square-root of an array, element-wise.
  `numpy.sqrt(x)`
  - x is the array with values whose square-roots are required.
- Return a matrix (or a 2D array) from an 1D array.
  `numpy.matrix(data)`
  - data is the 1D array.
  - Example: `numpy.matrix([1, 2, 3])`, output is `[[1, 2, 3]]`

- Insert a new axis that will appear at the axis position in the expanded array shape. `numpy.expand_dims(a,axis)`
  - a is the input array.
  - axis is an int or tuple of ints that represents poisition in the expanded axes where the new axis (or axes) is placed.

Answer:

```
M = np.dot(X_test, X_train.T)
te = np.square(X_test).sum(axis=1)
tr = np.square(X_train).sum(axis=1)
dists = np.sqrt(-2*M + np.matrix(tr) + np.matrix(te).T)
```

Alternative answer:

```
distances = np.sqrt(np.sum(np.square(np.expand_dims(X_test, 1) - X_train), axis=2))
```

```
distances = np.sqrt(np.sum(np.square(X_test[:,None] - X_train[None]), axis=-1))
```

Marking scheme:

- Case 0: Any for loop, list comprehension. # 0 point
- Case 1:
  (a) `M = np.dot(X_test, X_train.T)` # 2 points
  1 point if missing transpose
  (b) `te = np.square(X_test).sum(axis=1)` # 2 points
  `tr = np.square(X_train).sum(axis=1)` # 2 points
  - each square # 1 point
  - each sum with right axis # 1 point

  (c) `dists = np.sqrt(-2*M + np.matrix(tr) + np.matrix(te).T)` # 3 points
  - 1 point if shape doesn't match.
  - 2 points if np.matrix is missing, or attempt to modify the shape but wrong
  - if np.sqrt is missing,
    * if previous part isn't calculated correctly, 0 point for this part.
    * -1 point otherwise.

- Case 2:
  `distances = np.sqrt(np.sum(np.square(np.expand_dims(X_test, 1) - X_train), axis=2))`
  # 4 points
  (a) 4 points for the subtraction
    – 2 points if the shape modification is wrong.
    – 0 points if no shape modification at all.
  (b) 2 points for square
  (c) 2 points for sum;
    1 point if axis is missing or wrong
  (d) 1 point for sqrt (unlike Case 1, sqrt is worth 1 point here.)
  (e) -3 points if a^2–b^2 is used.

Remarks:

(a) If reshaped is used to change the shape of the solution, -1 point.

(b) If the program assume there's 4 train points or 2 test points, and the number of dimension is 2, -1 point or 0 point for the whole question.

(c) If Train, Test used instead of X_{train}, X_{test}, -1 point. If your program are terribly wrong (< 3 points) anyway, this mark wouldn't be deducted.

(d) If extra code which leads to wrong answer, -1 point.

(e) If the output shape is (num_{train}, num_{test}) instead of (num_{test}, num_{train}), -1 point.

(f) If *expand_dims*/*reshape* used without reassigning the variable, i.e. `a = a.reshape(*b.shape)`, -1 point.

(g) If missing shape, (e.g. $X_{train[0]}$ instead of $X_{train}.shape[0]$), – 0.5 point.

(h) Wrong spelling and syntax will not resulted in mark deduction, but 0.5 point will be deducted if you got full points.

(i) Index using i j assuming the existence of for loop gets 0 point.

Problem 3 [11 points] Conditional Probability and Bayes Classifier

(a) [2 points] Given the following probabilities:

- P(Good course | Desmond is in the course) = 0.5
- P(Good course | Desmond is not in the course) = 0.3
- P(Desmond in a randomly chosen course) = 0.1

What is P(Desmond is in the course | Not a good course)? If your answer is not an integer, write your answer in fraction form (use / to separate your numerator and denominator), e.g. 1/2.

Answer:

Let D be Desmond is in the course, G be a good course

$$\begin{aligned} P(D|NOT\ G) &= \frac{P(D, NOT\ G)}{P(Not\ G)} \\ &= \frac{P(Not\ G|D)P(D)}{P(Not\ G|D)P(D) + P(Not\ G|Not\ D)P(Not\ D)} \\ &= \frac{(1-0.5)\times 0.1}{(1-0.5)\times 0.1 + (1-0.3)\times(1-0.1)} \\ &= \frac{5}{68} \end{aligned}$$

Marking scheme:

- 2 points for giving the correct answer.

(b) [7 points] Suppose you are given the following set of data with three Boolean input variables A, B, C, and a single Boolean output variable D.

A	B	C	D
1	0	1	1
1	1	1	1
0	1	1	0
1	1	0	0
1	0	1	0
0	0	0	1
0	0	0	1
0	0	1	0

(i) According to the Naive Bayes classifier, what is $P(D = 1|A = 1, B = 1, C = 0)$? If your answer is not an integer, write your answer in fraction form (use / to separate your numerator and denominator), e.g. 1/2.

Answer:

$$P(D = 1|A = 1, B = 1, C = 0) = \frac{P(D = 1)P(A = 1|D = 1)P(B = 1|D = 1)P(C = 0|D = 1)}{\sum_{j=0}^{1} P(D = j)P(A = 1|D = j)P(B = 1|D = j)P(C = 0|D = j)}$$
$$= \frac{(4/8)(2/4)(1/4)(2/4)}{(4/8)(2/4)(2/4)(1/4) + (4/8)(2/4)(1/4)(2/4)} = \frac{1}{2}$$

Marking scheme:

- 1.75 points for giving the correct answer.

(ii) According to the Naive Bayes classifier, what is $P(D = 0|A = 1, B = 1)$? If your answer is not an integer, write your answer in fraction form (use / to separate your numerator and denominator), e.g. 1/2.

Answer:

$$P(D = 0|A = 1, B = 1) = \frac{P(D = 0)P(A = 1|D = 0)P(B = 1|D = 0)}{\sum_{j=0}^{1} P(D = j)P(A = 1|D = j)P(B = 1|D = j)}$$
$$= \frac{(4/8)(2/4)(2/4)}{(4/8)(2/4)(2/4) + (4/8)(2/4)(1/4)} = \frac{2}{3}$$

Marking scheme:

- 1.75 points for giving the correct answer.

(iii) According to the general Bayes classifier (i.e. without independence assumption), what is $P(D = 1|A = 1, B = 1, C = 0)$? If your answer is not an integer, write your answer in fraction form (use / to separate your numerator and denominator), e.g. 1/2.

Answer:

$P(D = 1|A = 1, B = 1, C = 0) = 0$ as there is no data with A = 1, B = 1, C = 0 and D = 1 in the dataset.

Marking scheme:

- 1.75 points for giving the correct answer.

(iv) According to the general Bayes classifier (i.e. without independence assumption), what is $P(D = 1|A = 1, B = 1)$? If your answer is not an integer, write your answer in fraction form (use / to separate your numerator and denominator), e.g. 1/2.

Answer:

$P(D = 1|A = 1, B = 1) = 1/2$ as number of data with A = 1, B = 1, D = 1 is 1, and number of data with A = 1, B = 1 is 2.

Marking scheme:

- 1.75 points for giving the correct answer.

(c) [2 points] The Naive Bayes algorithm selects the class c for an example x that maximizes $P(c|x)$. Suppose one of your classmates stated that it is equivalent to selecting the c that maximizes $P(x|c)$ under an assumption. What is the assumption that he has made?

Answer:

$P(c|x) = \frac{P(x|c)P(c)}{P(x)}$, so finding the c that maximizes $P(c|x)$ is equivalent to finding c that maximizes $P(x|c)$, if the prior $P(c)$ is uniform.

Marking scheme:

- 2 points for stating the assumption correctly.

Problem 4 [14 points] K-Nearest Neighbors

(a) [3 points] Consider a set of 5 training data given as $((x_1^{\texttt{Train}}, x_2^{\texttt{Train}}), c^{\texttt{Train}})$ values, where $x_1^{\texttt{Train}}$ and $x_2^{\texttt{Train}}$ are the two attribute values (positive integers) and $c^{\texttt{Train}}$ is the binary class label, A or B:

$$\{\ ((6,7),\text{A}),\ ((4,8),\text{B}),\ ((6,5),\text{B}),\ ((7,9),\text{A}),\ ((2,4),\text{A})\ \}$$

Classify a test example $(x_1^{\texttt{Test}}, x_2^{\texttt{Test}})$ with attribute values (4,7) using a KNN classifier with K = 3 and Manhattan distance defined by

$$\texttt{distance}(\mathbf{x}^{\texttt{Train}}, \mathbf{x}^{\texttt{Test}}) = \sum_{i=1}^{2} |x_i^{\texttt{Train}} - x_i^{\texttt{Test}}|$$

where $|\cdot|$ denote absolute value.

Complete the following table by filling in the computed Manhattan distance between each training data point and the test example. Determine the class label based on the results.

$x_1^{\texttt{Train}}$	$x_2^{\texttt{Train}}$	$c^{\texttt{Train}}$	**Distance**
6	7	A	
4	8	B	
6	5	B	
7	9	A	
2	4	A	

Answer:

$x_1^{\texttt{Train}}$	$x_2^{\texttt{Train}}$	$c^{\texttt{Train}}$	**Distance**
6	7	A	2
4	8	B	1
6	5	B	4
7	9	A	5
2	4	A	5

The predicted class label of the test example is B.

Marking scheme:

- 0.5 point for giving each correct distance value. 2.5 points in total.
- 0.5 point for giving the correct class label.

(b) [6 points] Judge whether each of the following student's claims is correct or not. Explain why.

(i) [3 points] A student claims that the results of a general KNN classifier that uses Euclidean distance will change if we multiply all attribute values of each training and test data point by 0.5.

Answer:

The claim is false, because K nearest neighbors will remain unchanged after multiplying all attribute values of each training and test data points by 0.5.

Marking scheme:

- 1 point for stating the claim is false.
- 2 points for giving the correct explanation.

Remark:

- 1 point given if stating the claim is correct but did mention that the change on distance will be a multiplication of constant to the old distance AND didn't state result of KNN classifier will change.
- 2 points given if stating the claim is correct but mention the change on distance will be a multiplication and state the result wont change.
- 0 point if correct deduction but draw the conclusion that result will change.
- 1 point for explanation if treat the question as asking multiply the values on Manhattan and answered correctly.
- 0 point for explanation if treat the question as asking comparison between Manhattan and euclidean distance.

(ii) [3 points] Another student claims that the classification accuracy of the training set will always increase if the value of K used in KNN classifier is incrementally increased from 1 to N, where N is the total number of training examples.

Answer:

The claim is false. A counterexample is as follows:
The training set accuracy when K = 1 will be 100%.
As K approaches the total number of training examples more and more examples influence the class, and eventually the class will always be the majority class in the training set.

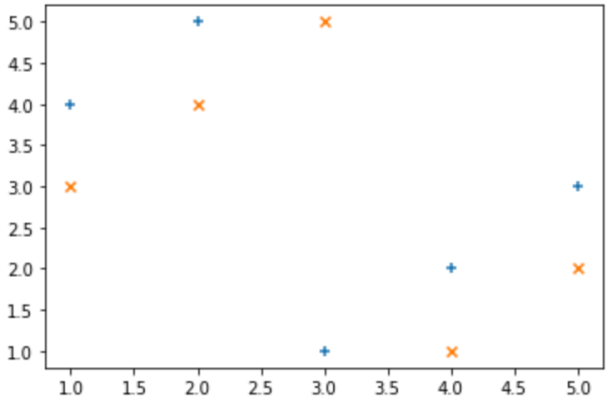
Marking scheme:

- 1 point for stating the claim is false.
- 2 points for giving the correct explanation.

Remarks:

- Give full mark if they misregard outliers as further neighbors.
- -1 point if they mention is due to outliers but didn't explain what is regarded as outliers.
- -2 points if they really mean outliers.
- -1 point if they only state larger k will cover more neighbors (this is just explaining what does a larger k means but not explaining why larger k can lower the accuracy) (*further neighbors is accepted but more neighbors is not)
- Other accept answers:
  i. Larger k include further neighbors which have low relevancy.
  ii. Underfitting
  iii. Giving a concrete situation that the claim is wrong.

(c) [5 points] Consider KNN using Euclidean distance on the following data set. Each point belongs to one of the two classes: + and x.



(i) [2 points] Perform 10-fold cross validation on the given data set (i.e. the 10 data points as shown in the figure), what is the validation error when using 1-nearest neighbor?

Answer:

Every point is misclassified. So, the validation error is 10/10.

Marking scheme:

- 2 points for giving the correct validation error.
  Note: Both 10/10 and 100% are accepted as the correct answers.

(ii) [3 points] Which of the following values of K leads to the minimum 10-fold cross validation error: 3, 5 or 9? What is the error for that K? If there is a tie, please elaborate.

Answer:

All 3 values of K mis-classify all of the points and have the same classification errors, 10/10.

Marking scheme:

- 2 points for stating all 3 values of K have the same classification errors.
- 1 point for giving the correct error.
  Note: Both 10/10 and 100% are accepted as the correct answers.

Problem 5 [12 points] K-Means Clustering

Given a 1-dimensional data set $\{0, 3, 6, 9, 27, 30\}$, use the K-means clustering algorithm and Euclidean distance to cluster the given points in the data set into 2 clusters. Assume c1 = 3 and c2 = 4 are chosen as the initial cluster centers.

(a) [4.5 points] Perform one iteration of K-means clustering by finding the Euclidean distance between each data point in the data set and the centroids, and assign each data point to the closest centroid according to the distance found. Fill in the following table with your computed values. If your answer is not an integer, write your answer in decimal form, e.g. 1.234.

Data Point	**Distance between the data point and c1 = 3**	**Distance between the data point and c2 = 4**	**Closest Centroid (c1 or c2?)**
0			
3			
6			
9			
27			
30			

Answer:

Data Point	**Distance between the data point and c1 = 3**	**Distance between the data point and c2 = 4**	**Closest Centroid (c1 or c2?)**
0	3	4	c1
3	0	1	c1
6	3	2	c2
9	6	5	c2
27	24	23	c2
30	27	26	c2

Marking scheme:

- 0.25 for giving each correct value. 4.5 points in total.

(b) [1.5 points] What are the values of c1 and c2 after one iteration of K-means? If your answer is not an integer, write your answer in decimal form, e.g. 1.234.

Answer:

$$c1 = \frac{0+3}{2} = 1.5$$
$$c2 = \frac{6+9+27+30}{4} = 18$$

Marking scheme:

- 0.75 for giving each correct centroid. 1.5 points in total.

(c) [4.5 points] Perform the second iteration of K-means clustering by finding the Euclidean distance between each data point in the data set and the computed centroids in part (b), and assign each data point to the closest centroid according to the distance found. Fill in the following table with your computed values. If your answer is not an integer, write your answer in decimal form, e.g. 1.234.

Data Point	**Distance between the data point and the c1 computed in part (b)**	**Distance between the data point and the c2 computed in part (b)**	**Closest Centroid (c1 or c2?)**
0			
3			
6			
9			
27			
30			

Answer:

Data Point	**Distance between the data point and the c1 computed in part (b)**	**Distance between the data point and the c2 computed in part (b)**	**Closest Centroid (c1 or c2?)**
0	1.5	18	c1
3	1.5	15	c1
6	4.5	12	c1
9	7.5	9	c1
27	25.5	9	c2
30	28.5	12	c2

Marking scheme:

- 0.25 for giving each correct value. 4.5 points in total.

(d) [1.5 points] What are the values of c1 and c2 after the second iteration of K-means? If your answer is not an integer, write your answer in decimal form, e.g. 1.234.

Answer:

$$c1 = \frac{0+3+6+9}{4} = 4.5$$
$$c2 = \frac{27+30}{2} = 28.5$$

Marking scheme:

- 0.75 for giving each correct centroid. 1.5 points in total.

Problem 6 [20 points] Perceptron

Given the following training dataset:

x_1	10	0	8	3	4	0.5	4	2
x_2	10	0	4	3	8	0.5	3	5
T	1	-1	1	-1	1	-1	1	1

(a) [8 points] Show the action of the perceptron algorithm for the above sequence of data points by completing the following table. Assume $\eta = 1$ and we start with the following initial weights and bias

$$w_1 = 1$$
$$w_2 = 1$$
$$\theta = 0$$

and use the following activation function.

$$f(z) = \begin{cases} 1 & z \geq 0 \\ -1 & otherwise \end{cases}$$

Updating rules:

$$\Delta w_i = \eta(T - O)x_i$$
$$\Delta\theta = \eta(T - O)$$
$$w_i = w_i + \Delta w_i$$
$$\theta = \theta + \Delta\theta$$

where $i \in \{1, 2\}$.

If your answer is not an integer, write your answer in decimal form, e.g. 1.234.

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	1	-	1	-	0

Answer:

x_1	x_2	T	O	Δw_1	w_1	Δw_2	w_2	$\Delta\theta$	θ
-	-	-	-	-	1	-	1	-	0
10	10	1	1	0	1	0	1	0	0
0	0	-1	1	0	1	0	1	-2	-2
8	4	1	1	0	1	0	1	0	-2
3	3	-1	1	-6	-5	-6	-5	-2	-4
4	8	1	-1	8	3	16	11	2	-2
0.5	0.5	-1	1	-1	2	-1	10	-2	-4
4	3	1	1	0	2	0	10	0	-4
2	5	1	1	0	2	0	10	0	-4

Marking scheme:

- 0.1 point for giving each correct value. 8 points in total.

(b) [2 points] According to the values in the table above, state whether the perceptron algorithm is converged in 1 epoch. If not, explain why.

Answer:

The algorithm is not converged in 1 epoch. Since there are changes of weights and biases.

Marking scheme:

- 1 point for stating the algorithm is not converged.
- 1 point for giving a correct explanation.

(c) [10 points] Write a Python program to verify your answers of Part (a). In your program, you need to define the following variables

- A 2D NumPy array, X, to store all the attribute values x_1, x_2, where the shape is (8,2)
- A 1D NumPy array, T,to store the target values, where the shape is (8,)
- A 1D NumPy array, W, to store the weights, where the shape is (2,)
- A float bias value, b.

and perform the required computations. Also, print the following sequence of values (in exact order) for each iteration:

$x_1 < s > x_2 < s > T < s > O < s > \Delta w_1 < s > w_1 < s > \Delta w_2 < s > w_2 < s > \Delta\theta < s > \theta < end >$

where $< s >$ refers to an empty space, and $< end >$ refers to an end of line character.

The following shows a line of sample output.

0 0 0 0 0 0 0 0 0 0

Remark: You cannot use any other libraries other than NumPy in your program.

Answer:

```
import numpy as np                          # 0.5 point

X = np.array([[10,10], [0,0], [8,4], [3,3], [4,8], [0.5,0.5], [4,3], [2,5]])  # 1 point
T = np.array([1,-1,1,-1,1,-1,1,1])          # 0.5 point
W = np.array([1,1], dtype=float)            # 1 point
b = 0.0                                     # 0.5 point

for i in range(X.shape[0]):                 # 1 point
  y = X[i].dot(W) + b                       # 1 point
  if y >= 0:                                # 0.5 point
    O = 1                                   # 0.5 point
  else:
    O = -1                                  # 0.5 point
  DW = (T[i] - O) * X[i]                    # 0.5 point
  W += DW                                   # 0.5 point
  Db = (T[i] - O)                           # 0.5 point
  b += Db                                   # 0.5 point

  print(X[i][0], X[i][1], T[i], O, DW[0], W[0], DW[1], W[1], Db, b)  # 1 point
```

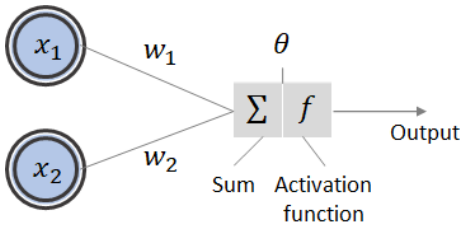
Remarks:

- -0.25 point for forgetting to specify `W = np.array([1,1], dtype=float)`
- Can also b = 0, Python will duck-type into float when needed.
- -0.25 point each for not using NumPy array. No overlap with W float penalty, since Python list supports duck-typing.
- -0.25 point each for wrong array shape.
- -0.25 point for forgetting bias in the output calculation.
- -0.25 point for minor print formatting errors.
- -0.25 point for incorrect print when output == target.
- -0.25 point each for miscellaneous syntax errors.
- -1 flat point for defining as class/function(s) but not calling.
- -10 floor point for using SKlearn.

Problem 7 [12 points] Perceptron and Multilayer Perceptron

(a) [4 points] Can we represent the given boolean function with a single neuron as shown below?

A	B	f(A,B)
0	0	0
0	1	0
1	0	1
1	1	0



If yes, show the possible weights, bias and activation function that can be used to compute the output of all the data points correctly. If not, explain why not in 1-2 sentences.

Answer:

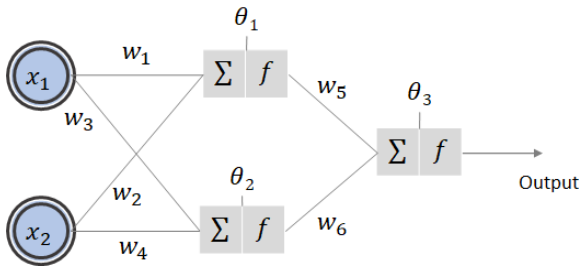
Yes, we can represent this function with a single neuron, since it is linearly separable. One set of possible weights and bias is: $w_1 = 1, w_2 = -1, \theta = -0.5$, and the activation function is

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & otherwise \end{cases}$$

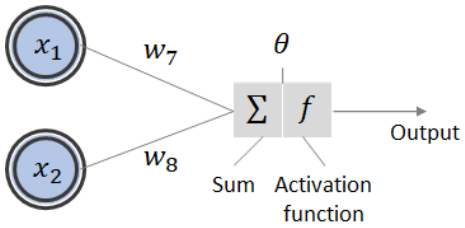
Marking scheme:

- 1 point for stating it is possible to represent the given boolean function with a single neuron.
- 1 point for showing the "standard" activation function.
- 2 points for correct values of w_1, w_2, and θ, given the activation function is the "standard" one.
- If the given activation function is not "standard", but the parameters are suitable, also get 2 points.
- 0 point for stating not possible to represent.

(b) [6 points] Suppose we have a neural network as shown below with $\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$, and linear activation function (i.e. $f(x) = Cx$, where C is a constant).



Can any function that is represented by the above network be represented by a single unit of artificial neural network in the following diagram? If so, detail the weights (w_7 and w_8), bias (θ), and the activation function $f(x)$. Otherwise, explain why not.



Answer:

Yes, the network can be represented by a single unit of artificial neural network by setting its weights, $w_7 = w_1w_5 + w_3w_6$, $w_8 = w_2w_5 + w_4w_6$, and the activation function $f(x) = C^2x$.

Marking scheme:

- 0.5 point for stating any function that is represented by the MLP can be represented by a single unit of artificial neural network.
- 1.5 points for showing each possible weight w_7, w_8. 3 points in total.
- 1 point for showing a possible bias value, i.e. 0.
- 1.5 for showing the activation function.

Remarks:

- If stating NO, -6 points. But among those NO's, if students put the expression OUTPUT=$C^2w_5(x_1w_1 + x_2w_2 + \theta_1) + C^2w6(x_1w_3 + x_2w_4 + \theta_2) + C\theta_3$, give 1 point.
- For those stating YES, if missing some value, deduct the score of that value.
- If the order of C is wrong, e.g. one C in w_7w_8 and no C in activation, or C^2 in w_7w_8 and one C in activation, -1 point.
- For other wrong values, -full mark for that values.
- If didn't plug in given values and the expression is wrong, even if the expression may evaluate to the same value as the answer, -full mark for that expression or -1 point if it's an order-of-C issue.
- If didn't plug in given values, e.g. θs=0, the same C for all units (some use $C_1C_2C_3$), but otherwise correct, -0.5 for each of θ & C.

(c) [2 points] Given a multilayer perceptron with 3 layers (input layer, hidden layer and output layer). The number of units in each of the these layers are 3, 4, 2. Assume each input or neuron is fully-connected. Calculate the number of trainable parameters of this network.

Answer:

$3 * 4 + 4 + 4 * 2 + 2 = 26$.

Marking scheme:

- 2 points for giving the correct answer.

---------- END OF PAPER ----------