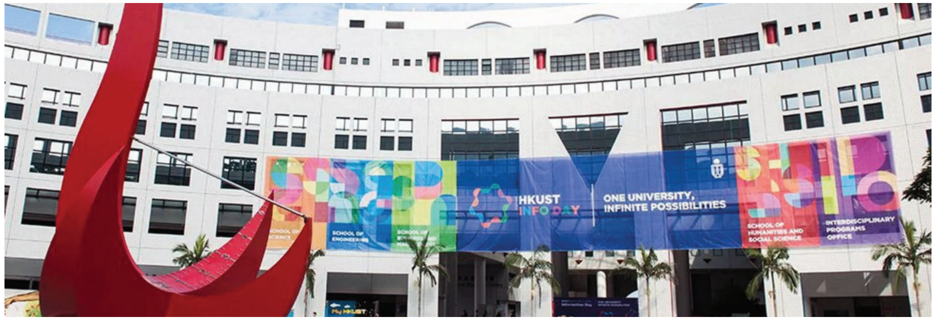

COMP 2012H Honors Object-Oriented Programming and Data Structures

Topic 16: Trees, Binary Trees, and Binary Search Trees

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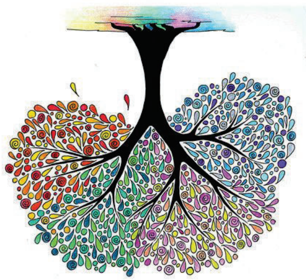
Part I

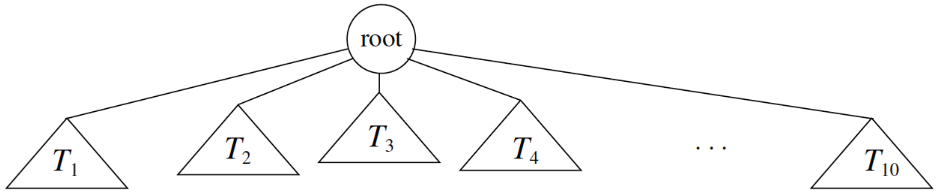
Tree Data Structure

Tree

- The linear access time of linked lists is prohibitive for large amount of data.
- Does there exist any simple data structure for which the average running time of most operations (search, insert, delete) is better than linear time?
- Solution: Trees!
- We are going to talk about
  - basic concepts of trees
  - tree traversal
  - (general) binary trees
  - binary search trees (BST)
  - balanced trees (AVL tree)



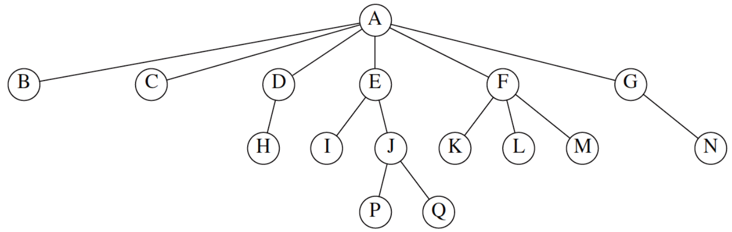
Recursive Definition of Trees



A tree T is a collection of nodes connected by edges.

- base case: T is empty
- recursive definition: If not empty, a tree T consists of
  - a root node r, and
  - zero or more non-empty sub-trees: $T_1, T_2, \ldots, T_k$

Tree Terminologies

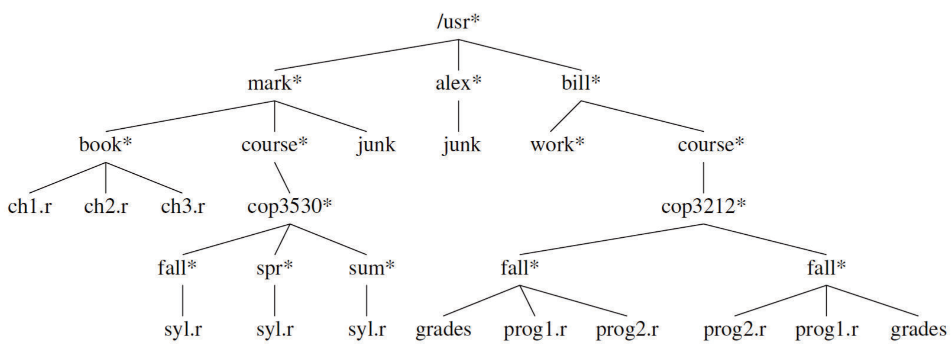


- Root: the only node with no parents
- Parent and child
  - every node except the root has exactly only 1 parent
  - a node can have zero or more children
- Leaves: nodes with no children
- Siblings: nodes with the same parent
- Path from node n_1 to n_k: a sequence of nodes $\{n_1, n_2, \ldots, n_k\}$ such that n_i is the parent of n_{i+1} for $1 \le i \le k-1$.

Tree Terminologies ..

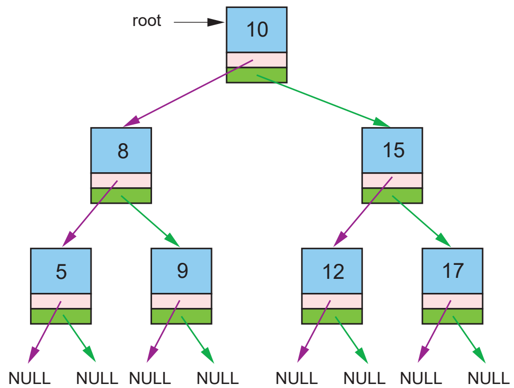
- Length of a path
  – number of edges on the path
- Depth of a node
  – length of the unique path from the root to that node
- Height of a node
  – length of the longest path from that node to a leaf
  – all leaves are at height 0
- Height of a tree
  = height of the root
  = depth of the deepest leaf
- Ancestor and descendant: If there is a path from n_1 to n_2
  – $n1$ is an ancestor of $n2$
  – $n2$ is a descendant of $n1$
  – if $n1 \neq n2$, proper ancestor and proper descendant

Example 1: Unix Directories in a Tree Structure

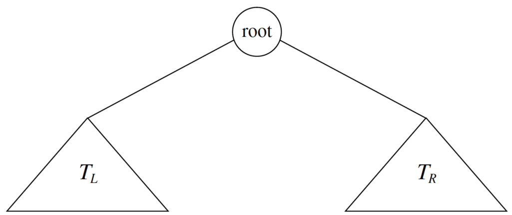


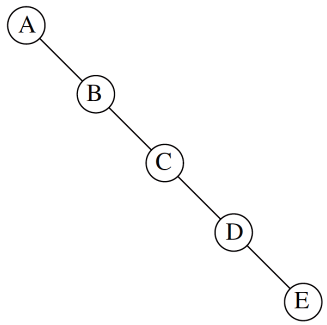
Part II

Binary Tree



Binary Trees





- Generic binary tree: A tree in which no node can have more than two children.
- The height of an 'average' binary tree with N nodes is considerably smaller than N.
- In the best case, a well-balanced tree has a height of order of $log\ N$.
- But, in the worst case, the height can be as large as $(N - 1)$.

A Typical Implementation of Binary Tree ADT

```
#include <iostream>     /* File: btree.h */
using namespace std;

template <class T> class BTnode
{
  public:
    BTnode(const T& x, BTnode* L = nullptr, BTnode* R = nullptr)
      : data(x), left(L), right(R) { }

    ~BTnode()
    {
        delete left;
        delete right;
        cout << "delete the node with data = " << data << endl;
    }

    const T& get_data() const { return data; }
    BTnode* get_left()  const { return left; }
    BTnode* get_right() const { return right; }

  private:
    T data;             // Stored information
    BTnode* left;       // Left child
    BTnode* right;      // Right child
};
```


Binary Tree: Inorder Traversal

```
/* File: btree-inorder.cpp
 *
 * To traverse a binary tree in the order of:
 *    left subtree, node, right subtree
 * and do some action on each node during the traversal.
 */

template <class T>
void btree_inorder(BTnode<T>* root,
     void (*action)(BTnode<T>* r)) // Expect a function on r->data
{
    if (root)
    {
        btree_inorder(root->get_left(), action);
        action(root);              // e.g. print out root->data
        btree_inorder(root->get_right(), action);
    }
}
```


Binary Tree: Preorder Traversal

```
/* File: btree-preorder.cpp
 *
 * To traverse a binary tree in the order of:
 *    node, left subtree, right subtree
 * and do some action on each node during the traversal.
 */

template <class T>
void btree_preorder(BTnode<T>* root,
     void (*action)(BTnode<T>* r)) // Expect a function on r->data
{
    if (root)
    {
        action(root);              // e.g. print out root->data
        btree_preorder(root->get_left(), action);
        btree_preorder(root->get_right(), action);
    }
}
```

Binary Tree: Postorder Traversal

```cpp
/* File: btree-postorder.cpp
 *
 * To traverse a binary tree in the order of:
 *    left subtree, right subtree, node
 * and do some action on each node during the traversal.
 */

template <class T>
void btree_postorder(BTnode<T>* root,
     void (*action)(BTnode<T>* r)) // Expect a function on r->data
{
    if (root)
    {
        btree_postorder(root->get_left(), action);
        btree_postorder(root->get_right(), action);
        action(root);              // e.g. print out root->data
    }
}
```


Example 2: Binary Tree Creation & Traversal

```cpp
#include "btree.h"      /* File: test-btree.cpp */
#include "btree-preorder.cpp"
#include "btree-inorder.cpp"
#include "btree-postorder.cpp"

template <typename T>
void print(BTnode<T>* root) { cout << root->get_data() << endl; }

int main() // Build the tree from bottom up
{   // Create the left subtree
    BTnode<int>* node5 = new BTnode<int>(5);
    BTnode<int>* node9 = new BTnode<int>(9);
    BTnode<int>* node8 = new BTnode<int>(8, node5, node9);
    // Create the right subtree
    BTnode<int>* node12 = new BTnode<int>(12);
    BTnode<int>* node17 = new BTnode<int>(17);
    BTnode<int>* node15 = new BTnode<int>(15, node12, node17);
    // Create the root node
    BTnode<int>* root = new BTnode<int>(10, node8, node15);

    cout << "\nInorder traversal result:\n"; btree_inorder(root, print);
    cout << "\nPreorder traversal result:\n"; btree_preorder(root, print);
    cout << "\nPostorder traversal result:\n"; btree_postorder(root, print);
    cout << "\nDeleting the binary tree ...\n"; delete root; return 0;
}
```


Example 3: Unix Directory Traversal

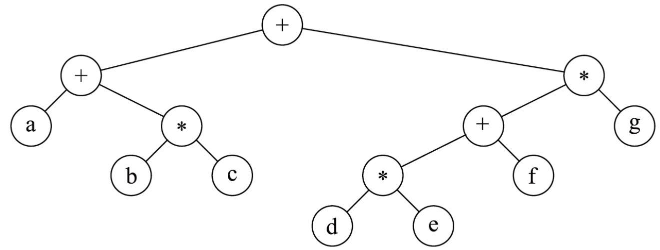
Preorder Traversal

```
/usr
    mark
        book
            ch1.r
            ch2.r
            ch3.r
        course
            cop3530
                fall
                    syl.r
                spr
                    syl.r
                sum
                    syl.r
        junk
    alex
        junk
    bill
        work
        course
            cop3212
                fall
                    grades
                    prog1.r
                    prog2.r
                fall
                    prog2.r
                    prog1.r
                    grades
```

Postorder Traversal

```
            ch1.r
            ch2.r
            ch3.r
        book
                    syl.r
                fall
                    syl.r
                spr
                    syl.r
                sum
            cop3530
        course
        junk
    mark
        junk
    alex
        work
                    grades
                    prog1.r
                    prog2.r
                fall
                    prog2.r
                    prog1.r
                    grades
                fall
            cop3212
        course
    bill
/usr
```


Example 4: Expression (Binary) Trees

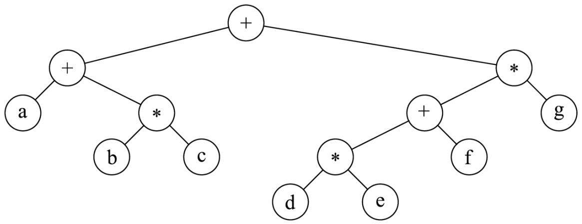


- Above is the tree representation of the expression:

$$(a + b * c) + ((d * e + f) * g)$$

- Leaves are operands (constants or variables).
- Internal nodes are operators.
- The operators must be either unary or binary.

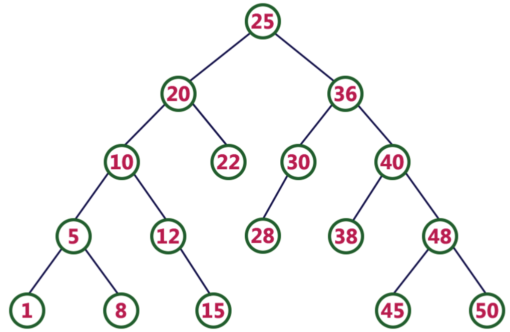
Expression Tree: Different Notations



- Preorder traversal: node, left sub-tree, right sub-tree.
  $\Rightarrow$ Prefix notation: $++a*bc*+*defg$
- Inorder traversal: left sub-tree, node, right sub-tree.
  $\Rightarrow$ Infix notation: $a+b*c+d*e+f*g$
- Postorder traversal: left sub-tree, right sub-tree, node.
  $\Rightarrow$ Postfix notation: $abc*+de*f+g*+$

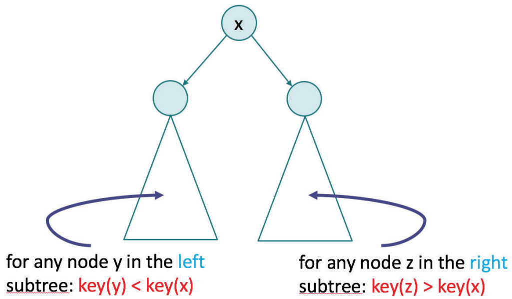
Part III

Binary Search Tree (BST)



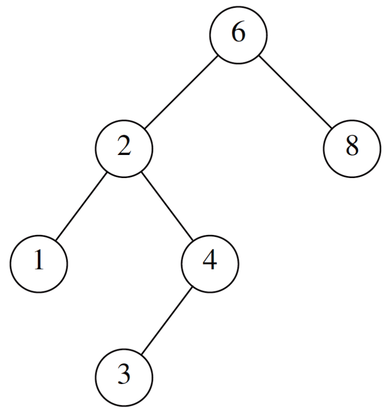
Properties of a Binary Search Tree

- BST is a data structure for efficient searching, insertion and deletion.
- BST property: For every node x
  - All the keys in its left sub-tree are smaller than the key value in node x.
  - All the keys in its right sub-tree are larger than the key value in node x.

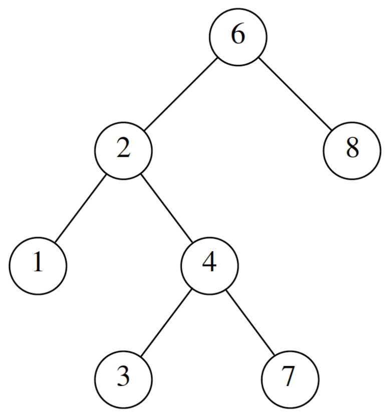


BST Example and Counter-Example

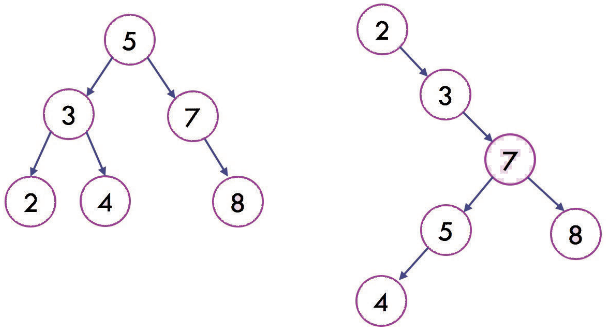
BST



Not a BST but a Binary Tree

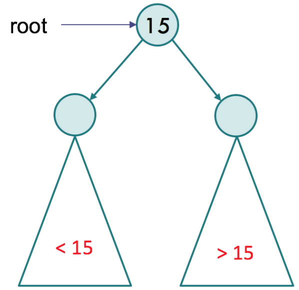


BSTs May Not be Unique



- The same set of values may be stored in different BSTs.
- Average depth of a node on a BST is order of log N.
- Maximum depth of a node on a BST is order of N.

BST Search

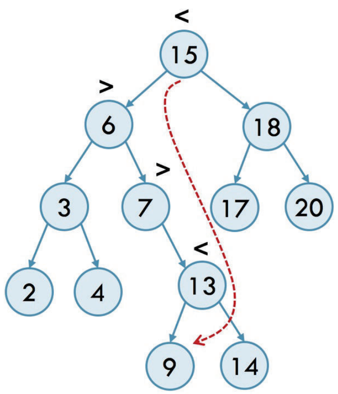


For the above BST, if we search for a value

- of 15, we are done at the root.
- $<$ 15, we would recursively search it with the left sub-tree.
- $>$ 15, we would recursively search it with the right sub-tree.

Example 5: BST Search

- Let's search for the value 9 in the following BST.



Compare	Action
9 vs. 15	continue with the left subtree
9 vs. 6	continue with the right subtree
9 vs. 7	continue with the right subtree
9 vs. 13	continue with the left subtree
9 vs. 9	eureka!

Our BST Implementation (different from textbook's) I

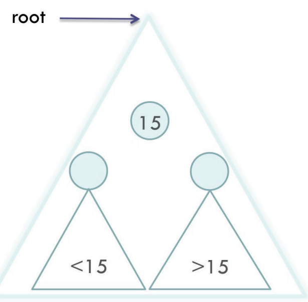
```
template <typename T> class BST /* File: bst.h */
{
  private:
    struct BSTnode      // A node in a binary search tree
    {
      T value;
      BST left;         // Left sub-tree or called left child
      BST right;        // Right sub-tree or called right child
      BSTnode(const T& x) : value(x) { } // A copy constructor for T
      // BSTnode(const T& x) : value(x), left(), right() { } // Equivalent
      BSTnode(const BSTnode& node) = default; // Copy constructor
      // BSTnode(const BSTnode& node)         // Equivalent
      //     : value(node.value), left(node.left), right(node.right) { }
      ~BSTnode() { cout << "delete: " << value << endl; }
    };
    BSTnode* root = nullptr;

  public:
    BST() = default;            // Empty BST
    ~BST() { delete root; }     // Actually recursive
```

Our BST Implementation (different from textbook's) II

```
    // Shallow BST copy using move constructor
    BST(BST&& bst) { root = bst.root; bst.root = nullptr; }

    BST(const BST& bst)  // Deep copy using copy constructor
    {
        if (bst.is_empty())
            return;

        root = new BSTnode(*bst.root);  // Recursive
    }

    bool is_empty() const { return root == nullptr; }
    bool contains(const T& x) const;
    void print(int depth = 0) const;
    const T& find_max() const; // Find the maximum value
    const T& find_min() const; // Find the minimum value

    void insert(const T&);     // Insert an item with a policy
    void remove(const T&);     // Remove an item
};
```


Our BST Implementation

- Our implementation really implements a BST as an object.
- It has a root pointing to a BST node which has
    - a value (of any type)
    - a left BST object: a sub-tree with values smaller than that of the root.
    - a right BST object: a sub-tree with values greater than that of the root.



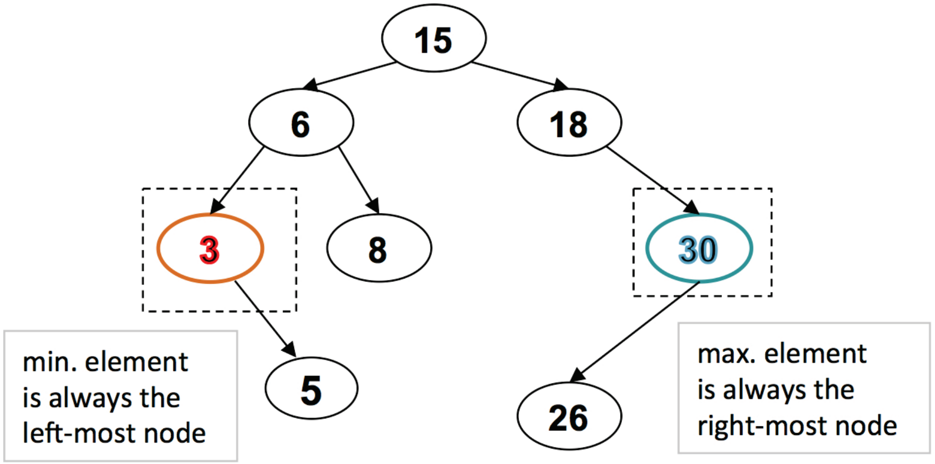
BST Code: Search

```
/* Goal: To check if the BST contains the value x.
 * Return: (bool) true or false
 * Time complexity: Order of height of BST
 */

template <typename T>         /* File: bst-contains.cpp */
bool BST<T>::contains(const T& x) const
{
    if (is_empty())           // Base case #1
        return false;

    if (root->value == x)     // Base case #2
        return true;

    else if (x < root->value) // Recursion on the left sub-tree
        return root->left.contains(x);

    else                      // Recursion on the right sub-tree
        return root->right.contains(x);
}
```


BST Code: Print by Rotating it -90 Degrees

```
/* Goal: To print a BST
 * Remark: The output is the BST rotated -90 degrees
 */

template <typename T>                /* File: bst-print.cpp */
void BST<T>::print(int depth) const
{
    if (is_empty())                  // Base case
       return;

    root->right.print(depth+1);      // Recursion: right sub-tree

    for (int j = 0; j < depth; j++) // Print the node value
        cout << '\t';
    cout << root->value << endl;

    root->left.print(depth+1);       // Recursion: left sub-tree
}
```

BST: Find the Minimum/Maximum Stored Value



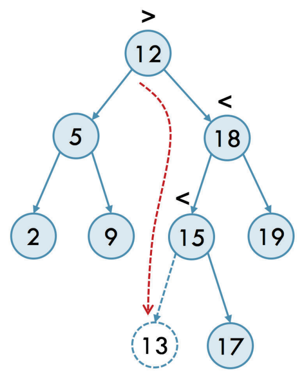
BST Code: Find the Minimum Stored Value

```
/* Goal: To find the min value stored in a non-empty BST.
 * Return: The min value
 * Remark: The min value is stored in the leftmost node.
 * Time complexity: Order of height of BST
 */

template <typename T>   /* File: bst-find-min.cpp */
const T& BST<T>::find_min() const
{
    const BSTnode* node = root;

    while (!node->left.is_empty()) // Look for the leftmost node
        node = node->left.root;

    return node->value;
}
```


BST Code: Find the Maximum Stored Value

```
/* Goal: To find the max value stored in a non-empty BST.
 * Return: The max value
 * Remark: The max value is stored in the rightmost node.
 * Time complexity: Order of height of BST
 */

template <typename T>   /* File: bst-find-max.cpp */
const T& BST<T>::find_max() const
{
    const BSTnode* node = root;

    while (!node->right.is_empty()) // Look for the rightmost node
        node = node->right.root;

    return node->value;
}
```


BST: Insert a Node of Value x

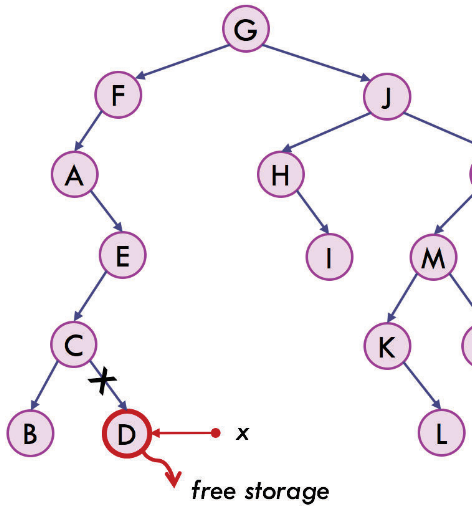


- E.g., insert 13 to the BST.
- Proceed down the tree as you would with a search.
- If x is found, do nothing (or update something).
- Otherwise, insert x at the last spot on the path traversed.
- Time complexity = Order of (height of the tree)

BST Code: Insertion

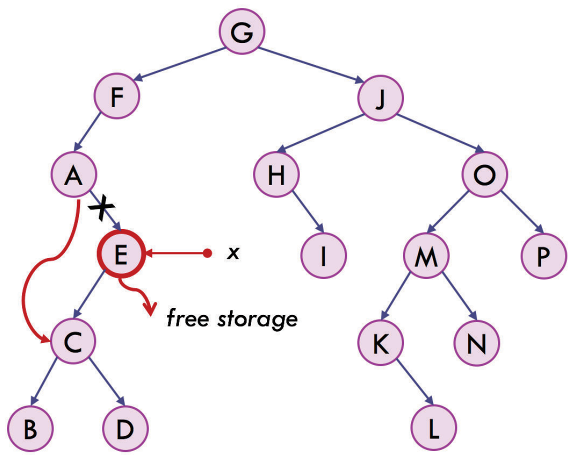
```
template <typename T>          /* File: bst-insert.cpp */
void BST<T>::insert(const T& x)
{
    if (is_empty())            // Find the spot
        root = new BSTnode(x);

    else if (x < root->value)
        root->left.insert(x);  // Recursion on the left sub-tree

    else if (x > root->value)
        root->right.insert(x); // Recursion on the right sub-tree

    else // This line is optional; just for illustration
        ;                      // x == root->value; do nothing
}
```


BST: Delete a Leaf



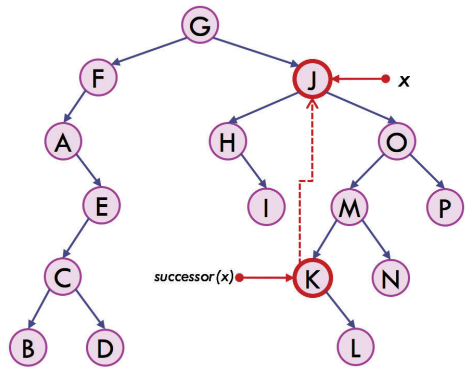
- Delete the leaf node immediately.

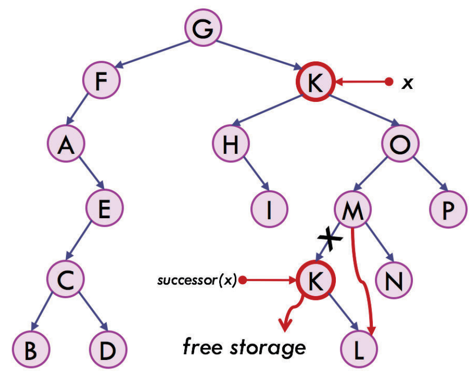
BST: Delete a Node with 1 Child



- Adjust a pointer from its parent to bypass the deleted node.

BST: Delete a Node with 2 Children

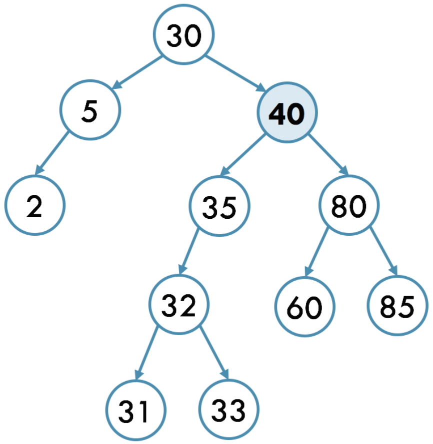




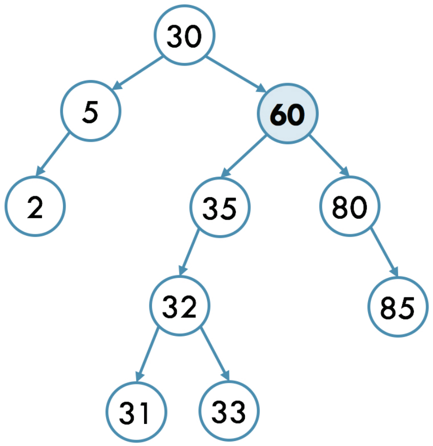
- You will have 2 choices: replace the deleted node with the
  - maximum node in its left sub-tree, or
  - minimum node in its right sub-tree (as in the above figure).
- Remove the max/min node depending on the choice above.

Example 6.1: BST Deletions

- Removing 40 from BST(a), replacing it with the min. value in its right sub-tree results in the BST(b).



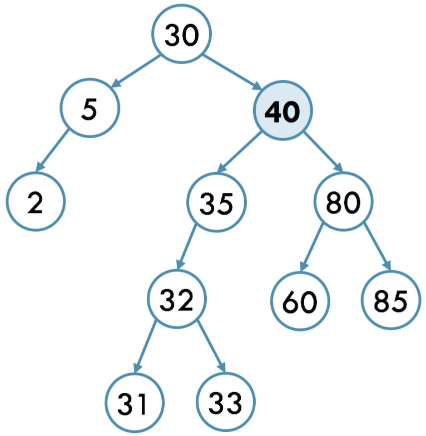
(a)



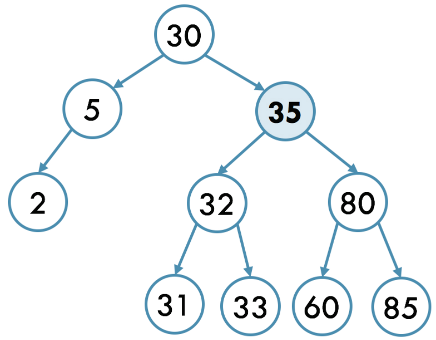
(b)

Example 6.2: BST Deletions

- Removing 40 from BST(a), replacing it with the max. value in its left sub-tree results in the BST(c).



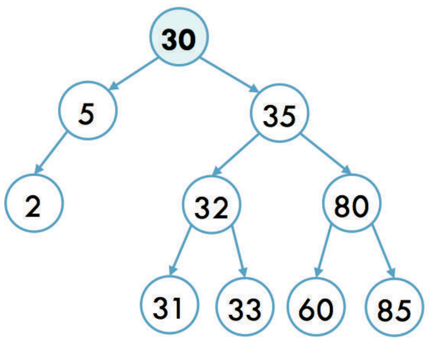
(a)



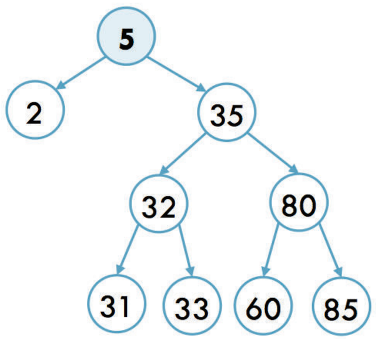
(c)

Example 6.3: BST Deletions

- Removing 30 from BST(c) and moving 5 from its left sub-tree result in BST(d).



(c)



(d)

BST Code: Deletion

```
template <typename T>           /* File: bst-remove.cpp */
void BST<T>::remove(const T& x) // leftmost item of its right subtree
{
    if (is_empty())             // Item is not found; do nothing
        return;

    if (x < root->value)        // Remove from the left subtree
        root->left.remove(x);
    else if (x > root->value)   // Remove from the right subtree
        root->right.remove(x);
    else if (root->left.root && root->right.root) // Found node has 2 children
    {
        root->value = root->right.find_min(); // operator= defined?
        root->right.remove(root->value); // min is copied; can be deleted now
    }
    else                        // Found node has 0 or 1 child
    {
        BSTnode* deleting_node = root; // Save the root to delete first
        root = (root->left.is_empty()) ? root->right.root : root->left.root;

        // Set subtrees to nullptr before removal due to recursive destructor
        deleting_node->left.root = deleting_node->right.root = nullptr;
        delete deleting_node;
    }
}
```

BST Testing Code I

```
#include <iostream>     /* File: test-bst.cpp */
using namespace std;
#include "bst.h"
#include "bst-contains.cpp"
#include "bst-print.cpp"
#include "bst-find-max.cpp"
#include "bst-find-min.cpp"
#include "bst-insert.cpp"
#include "bst-remove.cpp"

int main() {
    BST<int> bst;
    while (true) {
        char choice; int value;
        cout << "Action: d/f/i/m/M/p/q/r/s "
             << "(deep-cp/find/insert/min/Max/print/quit/remove/shallow-cp): ";
        cin >> choice;

        switch (choice) {
            case 'd': // Deep copy
            {
                BST<int>* bst2 = new BST<int>(bst);
                bst2->print(); delete bst2;
            }
                break;
```


BST Testing Code II

```
            case 'f': // Find a value
                cout << "Value to find: "; cin >> value;
                cout << boolalpha << bst.contains(value) << endl;
                break;
            case 'i': // Insert a value
                cout << "Value to insert: "; cin >> value;
                bst.insert(value);
                break;
            case 'm': // Find the minimum value
                if (bst.is_empty())
                    cerr << "Can't search an empty tree!" << endl;
                else
                    cout << bst.find_min() << endl;
                break;
            case 'M': // Find the maximum value
                if (bst.is_empty())
                    cerr << "Can't search an empty tree!" << endl;
                else
                    cout << bst.find_max() << endl;
                break;
            case 'p': // Print the whole tree
            default:
                cout << endl; bst.print();
                break;
```


BST Testing Code III

```
            case 'q': // Quit
                return 0;
            case 'r':
                cout << "Value to remove: "; cin >> value;
                bst.remove(value);
                break;
            case 's': // Shallow copy
            {
                BST<int> bst3 { std::move(bst) };
                bst3.print();
                bst.print();
            }
                break;
        }
    }
}
```


That's all!

Any questions?